The sign convention for refraction is different from the one for mirrors: object distances are counted as positive when the object is in front of the interface, but image distances are positive when the image is formed behind the interface. The radius of curvature follows the same convention as the image distances.

In the case of Fig. 4.1, the surface is convex, so the centre of curvature C lies to the right, and r is positive.

For the oblique ray, the incidence angle is θ and the refracted angle is ϕ. Then, by the exterior angle theorem, $\angle PCO = \theta - \alpha$ and $\angle PIC = \theta - \alpha - \phi$.

In the small angle approximation (see Appendix A), Snell's law becomes

$$n_1\theta = n_2\phi \tag{4.1}$$

and we can also approximate the angles as follows:

$$\alpha = \frac{x}{p} \tag{4.2}$$

$$\theta - \alpha = \frac{x}{r} \Rightarrow \theta = \frac{x}{p} + \frac{x}{r} \tag{4.3}$$

$$\theta - \alpha - \phi = \frac{x}{i} \Rightarrow \phi = \frac{x}{r} - \frac{x}{i} \tag{4.4}$$

and substituting θ and ϕ in Snell's law, we get after cancelling x

$$\frac{n_1}{p} + \frac{n_1}{r} = \frac{n_2}{r} - \frac{n_2}{i} \tag{4.5}$$

which can be rearranged more meaningfully to

$$\frac{n_1}{p} + \frac{n_2}{i} = \frac{n_2 - n_1}{r} \tag{4.6}$$

If the light is passing from air of refractive index $n_1 = 1$ to glass of index $n_2 = n$, equation 4.6 becomes

$$\frac{1}{p} + \frac{n}{i} = \frac{n-1}{r} \tag{4.7}$$

4.3 A lens

4.3.1 Locating the image

In a lens, there are two consecutive refractions, one from air to glass, and then from the glass back into the air. Figure 4.2 shows the process. Applying eq. (4.6) to the first refraction, we get